Welcome to the Putnam Seminar! Remember to join the HMC Google Group for the Putnam Seminar. This will keep you informed about the Seminar and the Putnam Exam on Saturday, December 4, 2021. To do this, send an email to: `putnam-l+subscribe@g.hmc.edu`

Putnam Practice Problems # 2

Tuesday, September 14, 2021

Polynomials

B1: For which real values of p and q are the roots of the polynomial $x^3 - px^2 + 11x - q$ three successive integers? Give the roots in these cases. (Barbeau, Klamkin, and Moser)

B2: (a) For which integers n is $n^3 + 1000$ divisible by $n + 10$?

(b) What is the largest integer n such that $n^3 + 100$ is divisible by $n + 10$?

B3: (a) Determine all polynomials $P(x)$ such that $P(x + 1) = P(x) + 1$ and $P(0) = 0$.

(b) Determine all polynomials $P(x)$ such that $P(x^2 + 1) = [P(x)]^2 + 1$ and $P(0) = 0$.

(Putnam, 1971)

B4: Find all real numbers c such that the graph of $x^4 + 9x^3 + cx^2 + 9x + 4$ meets some line in four distinct points. (Putnam, 1994)

And for a little bit of variety...

B5: Find a nine-digit number made up of $1, 2, 3, 4, 5, 6, 7, 8, 9$ in some order such that when digits are removed one at a time from the right the number remaining is divisible in turn by $8, 7, 6, 5, 4, 3, 2, 1$. (Hess)

Hints:

1. The sum of the roots is... the product of the roots is... and what does the coefficient of x represent in terms of the roots?
2. A little polynomial division goes a long way in this problem.
3. What values of P can you compute?
4. When can a line meet a quartic in four points?