(Mock) GRE Math Subject Test

Created by: Mohamed Omar & Ivan Ventura
Harvey Mudd College

1. $\frac{\partial}{\partial x}\frac{\partial}{\partial y}\left(2xy^2 - \sin(\sqrt{1+y^2})e^{-y}\right)$ evaluated at $x = -1$, $y = 1$ is

   A. -4

   B. -2

   C. 0

   D. 2

   E. 4

2. The statement " $\neg$ P $\Longrightarrow$ (Q or R) " is logically equivalent to

   A. P $\Longrightarrow$ ($\neg$ Q and $\neg$ R)

   B. P $\Longrightarrow$ ($\neg$ Q or $\neg$ R)

   C. ($\neg$ Q or $\neg$ R) $\Longrightarrow$ P

   D. (Q and R) $\Longrightarrow$ P

   E. ($\neg$ Q and $\neg$ R) $\Longrightarrow$ P

3. If A is a 3×5 matrix that has rank 2, the maximum number of linearly independent vectors that can be chosen from the null space of A is

   A. 0

   B. 1

   C. 2

   D. 3

   E. 4

4. The units digit of the integer 3^{5^7} is

   A. 1

   B. 3

   C. 7

   D. 9

   E. none of the above

5. If $f(x) = x \cdot |x|$ then

I) f is continuous on all of $\mathbb{R}$

II) f is not differentiable at $x = 0$

III) f is odd

A. I only

B. I & II

C. I & III

D. II & III

E. I, II & III

6. The number of real solutions to the equation $e^x = 9 - x^3$ is

A. 0

B. 1

C. 2

D. 3

E. infinite

7. The directional derivative of $f(x, y, z) = xyz$ at the point $(1, -1, 1)$ in the direction $\vec{v} = (2, 0, 0)$ is

   A. -2

   B. -1

   C. 1

   D. 2

   E. $\dfrac{1}{\sqrt{3}}$

8. Let C be the closed curve of radius 3 centered at the origin oriented counterclockwise. The value of $\oint_C (2y^2 - 2x)\ dx + 3x\ dy$ is

   A. -9π

   B. 9π

   C. 18π

   D. 27π

   E. 45π

9. Suppose x and y are integers, and $8x - 5y$ is divisible by 7. Which of the following must also be divisible by 7?

   A. $-6x + 2y$

   B. $-6x + 3y$

   C. $-5x + 2y$

   D. $-5x + 3y$

   E. $-5x - 2y$

10. A fair coin is flipped 150 times. The probability that heads appears at most 62 times is closest to

    A. 0.02

    B. 0.05

    C. 0.1

    D. 0.17

    E. 0.34

11. $\iint_D \frac{1}{4+x^2+y^2}\, dx\, dy$ where D is the region in the xy-plane enclosed by a circle of radius 2 centered at the origin is

A. $\pi \cdot \log(\sqrt{2})$

B. $\pi \cdot \log(2)$

C. $\pi \cdot \arctan(4)$

D. $\pi \cdot \arctan(16)$

E. $\frac{\pi^2}{8}$

12. Let B be a 3×3 matrix such that $B^2 = B$. Which of the following statements must be true:

A. B is invertible

B. $\det(B) = 0$

C. $\det(B^5) = \det(B)$

D. 1 is an eigenvalue of B

E. B is upper triangular

13. Let $f : [0, 1] \to [0, 1]$ be an invertible continuous function and suppose $f(0) = 0$ and $f(1) = 1$. Which of the following must equal $\displaystyle\int_0^1 f^{-1}(x)\ dx$?

A. $\dfrac{1}{2}$

B. $\displaystyle\int_0^1 f(x)\ dx$

C. $1 - \displaystyle\int_0^1 f(x)\ dx$

D. $\left(\displaystyle\int_0^1 f(x)\ dx\right)^{-1}$

E. none of the above

14. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and let $S = \{f(c) \ : \ 0 < c < 1\}$. Which of the following must be true:

I) S is connected.

II) S is open.

III) S is bounded.

A. I only

B. I & II

C. I & III

D. II & III

E. I, II & III

15. If C is the closed curve $\left\{3e^{i\theta} \ : \ 0 \leq \theta < 2\pi\right\}$, then $\oint_C \frac{1}{(z-2)(z+4)^2}\, dz$ is

A. 0

B. $2\pi i$

C. $\frac{\pi i}{2}$

D. $\frac{\pi i}{18}$

E. $\frac{\pi i}{36}$

16. The number of finite groups (up to isomorphism) which contain exactly 3 elements of order 3 is

A. 0

B. 1

C. 2

D. 5

E. infinite

17. $\lim_{x \to 0} \dfrac{1 - \frac{1}{2}x^2 - \cos(x)}{x^4} =$

A. $-\dfrac{1}{24}$

B. $-\dfrac{1}{2}$

C. $-\dfrac{1}{12}$

D. $\dfrac{1}{24}$

E. $-\dfrac{1}{2}$

18. Suppose X is a random variable defined on the nonnegative integers. Which of the following can not be a probability density function for X

A. $P(X = k) = \dfrac{3^k}{e^3 \cdot k!}$ for any $k \geq 0$

B. $P(X = k) = \begin{cases} \dfrac{k}{55} & \text{if } k < 10 \\ 0 & \text{if } k \geq 10 \end{cases}$

C. $P(X = k) = \dfrac{2}{3^{k+1}}$ for any $k \geq 0$

D. $P(X = k) = \begin{cases} \dbinom{12}{k} \dfrac{9^k}{10^{12}} & \text{if } k \leq 12 \\ 0 & \text{if } k > 12 \end{cases}$

E. $P(X = k) = \dfrac{1}{\sqrt{k+1}} - \dfrac{1}{\sqrt{k+2}}$ for any $k \geq 0$

19. Which of the following is true about the function

$$f(x, y) = 4 + x^3 - 3xy + y^3$$

I) It has one relative maximum

II) It has one relative minimum

III) It has one saddle point

A. I only

B. II only

C. I & II

D. I & III

E. II & III

20. Let A be an $n \times n$ real symmetric matrix with nonnegative eigenvalues. Which of the following is not necessarily true about A?

A. $x^T Ax$ defines an inner product on $\mathbb{R}^n$

B. There is a symmetric matrix B such that $A = B^2$

C. There is an orthonormal basis of $\mathbb{R}^n$ consisting of eigenvectors of A

D. The null space of A is orthogonal to the column space of A

E. A^3 has nonnegative eigenvalues

21. Which of the following capital letters has a non-abelian fundamental group?

A B C D E

A. A

B. B

C. C

D. D

E. E

22. Let $a_1, a_2, \ldots, a_n, , \ldots$ be a sequence of nonnegative real numbers for which $\sum_{n=1}^{\infty} a_n$ converges. Which of the following series must converge?

I) $\sum_{n=1}^{\infty} (-1)^n a_n$

II) $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$

III) $\sum_{n=1}^{\infty} \sqrt{a_n}$

A. I only

B. II only

C. I & II

D. I & III

E. II & III